A PERTURBATION THEORY FOR MICROSTRIP PROPAGATION

R. E. Collin and B. Kretch

Department of Electrical Engineering
and Applied Physics
Case Western Reserve University
Cleveland, Ohio 44106

ABSTRACT

A perturbation theory for propagation on an open microstrip transmission line is developed by expanding the field and the propagation constant in a power series in k_0, the free space wave number. This theory only requires the use of two relatively simple static Green's functions.

Introduction

A dynamic full wave analysis of the standard unshielded microstrip transmission line is very complex because of the coupling of TE and TM waves and the complicated structure of the required Green's dyadic function. On the other hand at the lower frequencies the zero'th order quasi-static solution is known to be quite accurate. It is therefore apparent that a power series solution in powers of the radian frequency ω should provide a much needed extension of the available quasi-static solution along with a description of the dispersive properties of the line and the field distribution in the cross section of the line. In this paper such a perturbation theory is developed. It establishes the usual quasi-static results as the zero frequency limit of the exact solution as well as providing a systematic scheme to find the higher order corrections.

The Perturbation Expansion

The theory is developed by using the vector potential function $\vec{A}$ and scalar potential Φ in the Lorentz gauge. The vector potential function is assumed to have the form $\vec{A}(x,y)e^{-j\beta z}$ where β is the unknown propagation constant. The function $\vec{A}(x,y)$ is expanded as a perturbation series

$$\vec{A}(x,y) = \vec{A}_0 + \eta\vec{A}_1 + \eta^2 A_2 + \ldots \quad (1)$$

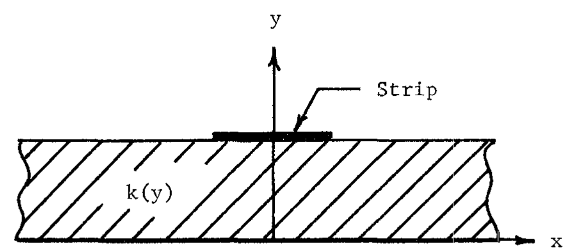
The scalar potential is represented by the series

$$\Phi(x,y) = \Phi_0 + \eta\,\Phi_1 + \eta^2\Phi_2 + \ldots \quad (2)$$

Since $\beta(\omega)$ must be an odd function of ω it is expanded as

$$\beta = \eta\beta_1 + \eta^3\beta_3 + \ldots \quad (3)$$

In a similar way the fields $\vec{E}$ and $\vec{H}$ and the current $\vec{J}$ and charge ρ on the strip (see Fig. 1) are expanded as perturbation series. The perturbation parameter η is associated with ω so that $k_0^2 = (\omega^2/c^2)$ is replaced by $\eta^2 k_0^2$.



Ground Plane

Fig. 1

The equations that determine the fields are

$$[\nabla_t^2 + (\eta^2\kappa(y)\,k_0^2 - \beta^2)]\,\vec{A} = -\,\mu_0\,\vec{J} \quad (4a)$$

$$[\nabla_t^2 + (\eta^2\kappa(y)\,k_0^2 - \beta^2)]\,\Phi = -\,\rho/\varepsilon_0 \quad (4b)$$

$$j\eta\,\omega\,\mu_0\varepsilon_0\kappa(y)\,\vec{E} = \eta^2\kappa(y)k_0^2\,\vec{A} + \nabla_t\nabla_t\cdot\vec{A} - j\beta\nabla_t A_z - j\beta\vec{a}_z\nabla_t\cdot\vec{A} - \beta^2\vec{a}_z A_z \quad (4c)$$

$$\mu_0\,\vec{H} = \nabla_t \times \vec{A} - j\beta\vec{a}_z \times \vec{A} \quad (4d)$$

$$\nabla_t\cdot\vec{A} - j\beta A_z = -\,j\eta\omega\kappa(y)\,\varepsilon_0\mu_0\,\Phi \quad (4e)$$

along with appropriate boundary conditions on the ground plane, on the strip, and at the air-dielectric interface.

An alternative formulation could be based on the use of the vector and scalar potentials in the Coulomb gauge. This has the disadvantage of requiring a three component vector potential function that satisfies the condition $\nabla\cdot\vec{A} = 0$. By using the Lorentz gauge there are fewer unknown functions to relate together at each level of the approximation.

0149-645X/85/0000-0561

The lowest order approximation requires a solution for A_{0z}, Φ_0 where

$$\nabla_t^2 \; A_{0z} = - \mu_0 \; J_{0z} \tag{5a}$$

$$\nabla_t^2 \Phi_0 = - \frac{\rho_0}{\varepsilon_0} \tag{5b}$$

The boundary conditions impose on these potentials the conditions

A_{0z}, Φ_0 are zero on the ground plane (6a)

A_{0z}, Φ_0 are constant on the strip (6b)

A_{0z}, $\frac{\partial A_{0z}}{\partial y}$, Φ_0, and $\kappa(y) \frac{\partial \Phi_0}{\partial y}$ are continuous

at the air-dielectric interface (6c)

$\omega A_{0z} = \beta_1 \; \Phi_0$ on the strip (6d)

The above equations are solved using two static scalar Green's functions that satisfy

$$\nabla_t^2 \; G_i = - \delta(x-x') \; \delta(y-y'), \; i - 1, \; 2 \tag{7}$$

where G_1 and $\frac{\partial G_1}{\partial y}$ are continuous at the air-dielectric interface and $G_1 = 0$ on the ground plane while G_2 and $\kappa(y)\frac{\partial G_2}{\partial y}$ are continuous at the interface and $G_2 = 0$ on the ground plane also. The solution for G_1 is obtained as a solution to a simple image problem. The solution for G_2 is developed as an infinite image series.

From the solution of the above system it is found that $\beta_1 = \omega(L_0 C_0)^{1/2}$ where L_0 and C_0 are the expected static inductance and capacitance per unit length.

At the next level of approximation A_{2z}, A_{1x} and Φ_2 must be found where

$$\nabla_t^2 \; A_{2z} = - \mu_0 \; J_{2z} + [\beta_1^2 - \kappa(y) k_0^2] \; A_{0z} \tag{8a}$$

$$\nabla_t^2 \; \Phi_2 = - \frac{\rho_2}{\varepsilon_0} + [\beta_1^2 - \kappa(y) \; k_0^2] \; \Phi_0 \tag{8b}$$

along with appropriate boundary conditions that couple A_{2z}, A_{1x} and Φ_2 and involving the next higher order term β_3 in the expansion of β. The equations can be solved using the same Green's functions G_1 and G_2.

Numerical results for β_1 and β_3 for typical microstrip lines will be given in the paper as well as a more complete development of the theory and the relevant boundary conditions. One major advantage of this theory is that only two relatively simple scalar Green's functions are required.

Zero'th Order Solution

For the zero'th order solution the current on the strip is approximated by the following expression:

$$J_{0z} = \frac{I_0 + I_1 \; x^2 + I_2 x^4 + I_3 x^6}{\sqrt{1 - x^2}} \tag{9}$$

and a similar form is used for ρ_0. In the above equation the length unit has been chosen so that the strip is 2 units wide. It is found that I_2 and I_3 are very small and that I_1 accounts for the presence of the ground plane.

On the strip we can write

$$A_{0z} = I \; L_0 \tag{10a}$$

$$\Phi_0 = \frac{Q}{C_0} = V \tag{10b}$$

where I is the total current on the strip, Q is the total charge on the strip, and L_0 and C_0 are the static inductance and capacitance per unit length. The vanishing of E_{0z} on the strip imposes the condition (6d) on the potentials. This condition gives

$$\omega L_0 \; I = \beta_1 \; \Phi_0 = \beta_1 \; V \tag{11}$$

The continuity equation relating current and charge requires that

$$\beta_1 \; I = \omega \, Q \tag{12}$$

From (10b) and (12) we obtain

$$\beta_1 \; I = \omega \; C_0 \; V$$

which, when combined with (11) gives

$$\beta_1^2 = \omega^2 \; C_0 \; L_0 \text{ and } V = (L_0/C_0)^{1/2} \; I$$

These are the expected quasi-static results. Note, however, that even in the zero'th order approximation the theory does not require that the axial electric field be zero.

When the higher order approximations are developed it is found that A_{0x}, Φ_1, A_{1z} are not required. Only the even numbered terms in A_z, Φ and the odd numbered terms in A_x are coupled together.

This work was supported by NASA-Lewis Research Center, Cleveland, Ohio, under Grant NCC3-29